

Motion Analysis Classwork

1.

A particle moves on the x -axis so that its position at any time $t \geq 0$ is given by $x(t) = 2te^{-t}$.

- (a) Find the acceleration of the particle at $t = 0$.
- (b) Find the velocity of the particle when its acceleration is 0.
- (c) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

2.

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

- (a) Find $f'(x)$ and $f''(x)$.
- (b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k , determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.
- (c) For a certain value of the constant k , the graph of f has a point of inflection on the x -axis. Find this value of k .

3.

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \quad \text{and} \quad f''(0) = 3.$$

- (a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show the work that leads to your answers.
- (b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.